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Certain Application of GERT Network Analysis in Statistical Quality Control

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Abstracts

Graphical Evaluation and Review Technique commonly known as GERT, is a network analysis technique effectively used in project management that allows probabilistic treatment of both network logic and estimation of activity duration. The technique was first discovered in 1966 by Dr. Alan B. Pritsker of Purdue University. Compared to other techniques, GERT is rarely used in complex systems. The GERT approach, addresses the majority of limitations associated with PERT/ CPM technique. The fundamental drawback associated with the GERT technique is the complex programme required to model the GERT system. Development in GERT includes Q- GERTS allowing the user to consider queuing within the system. Graphical Evaluation and Review Technique has been applied in modeling of sampling plans and promises to be value in encouraging statistical quality control. In this paper we analyze a sampling plan using GERT technique.

Keywords: Sampling plan, Acceptance Sampling, Acceptance, Rejection, GERT Analysis, Average Out going Quality, Expected amount of inspection..

Introduction

One of the major areas of Statistical Quality Control is Acceptance Sampling. Acceptance Sampling is a methodology that deals with procedures, by which decision to accept or reject are based on the inspection samples[2]. According to Duncan (1986) acceptance sampling is likely to be used under the following conditions:

When the cost of inspection is high and loss arising out of passing non-conforming unit is not great, then in some cases it is possible no inspection will be carried which will be the cheapest plan.

When 100% inspection is fatigue a carefully worked out sampling plan will produce good or better results hundred percent may not mean 100% perfect quality the percentage on non-conforming items passed may be higher than a scientifically designed sampling plan.

When inspection is destructive the sampling inspection must be employed.

GERT7[7] is a technique for the analysis of a class of networks which have the following characteristics: (i) a probability that a branch of the network is indeed part of a realization of the network [8]; and (ii) an elapsed time or time interval associated with the branch if the branch is part of the realization of the network. Such networks will be referred to as stochastic networks and consist of a set of branches and nodes. A realization of a network is a particular set of branches and nodes which describe the network for one experiment. If the time associated with a branch is a random variable, then a alization also implies that a fixed time has been selected for each branch. GERT will derive both probability that a node is realized and the conditional moment generating function (M.G.F) of the elapsed time required to tranverse betweenany two nodes.

STEPS in applying GERT

The foregoing material described th qualitative aspects of GERT. Basically, the steps employed in applying GERT are:

1. Convert a qualitative description of a system or problem to a model in network form;
2. Collect the necessary data to describe the branches of the network;
3. Obtain an equivalent one-branch function between two nodes of thr network;
4. Convert the equivalent function into the following two performance measures of the network;
   a. The probability that a specific node is realized; and
   b. The M.G.F of the time associated with an equivalent network;

Continuous sampling plan -1

The operating procedure of the CSP-1 plan as stated by Dodge[5] is as follows:

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(a) At the outset, inspect 100% of the units consecutively as produced and continue such inspection until i units in succession are found clear of defects.
(b) When i units in succession are found clear of defects, discontinue 100% inspection, and inspect only a fraction f of the units, selecting individual units one at a time from the flow of product, in such a manner as to ensure an unbiased sample.
(c) If a sample unit is found defective [1], revert immediately to a 100% inspection of succeeding units and continue until again i units in succession are clear of defects, as in step (a).
(d) Correct or replace with good units, all defective units found.
Thus, the CSP-1 plans are characterized by two parameters i and f.

In this paper, however, reference will be made to a sample size parameter only. The sample size n associated with a branch is characterized by the moment generating function (*mgf*) of the form $M_n(\theta) = \sum_n \exp(n\theta)\, f(n)$, where $f(n)$ denotes the density function of n and θ is any real variable. The probability φ that the branch is realized is multiplied by the *mgf* to yield the *W*-function such that

$$W(\theta) = \phi M_n(\theta)$$

The *W*-function is used to obtain the information on the relationship which exists between the nodes.

Graphical evaluation & review technique - analysis of plan

The possible states of the CSP-1 inspection[3] system described can be defined as follows:
S_0 : Initial state of the plan.
$S_1(k)$: State in which $k(= 1, 2..., i)$ preceding units are found clear of defects during 100% inspection.
S_P : Initial state of partial inspection.
S_2 : State in which a unit is not inspected (i.e. passed) during sampling inspection.
S_{PA} : State in which a unit is found free of defects during partial (sampling) inspection.
S_{PR} : State in which a unit is found defective during partial inspection.
S_A : State in which current unit is accepted.
S_R : State in which current unit is rejected.

The above states enable us to construct GERT network representation of the inspection system as shown in Fig. (1) and (2). Suppose that the process is in statistical control, so that the probability of any incoming unit being defective is (p) and the probability of any unit being non-defective is $q = 1 - p$. The *W*-functions from the initial node S_0 to the terminal nodes S_{PA} and S_{PR} are respectively found as

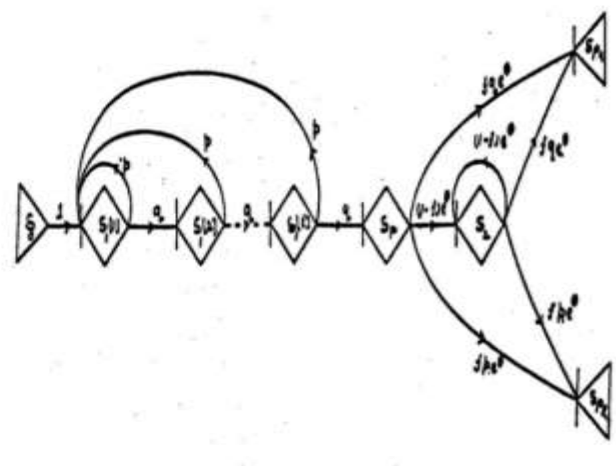
$$\boldsymbol{W_{LA}(\theta)} = \frac{\boldsymbol{f q^i[1-(1-f)e^\theta]e^\theta + f(1-f)q^i e^{2\theta}}}{\boldsymbol{1-[(1-q^i)+(1-f)e^\theta]+(1-f)((1-q^i)e^\theta}}$$

…………(3.1)

and

$$\boldsymbol{W_{LR}(\theta)} = \frac{\boldsymbol{f p q^i[1-(1-f)e^\theta]e^\theta + f p(1-f)q^i e^{2\theta}}}{\boldsymbol{1-[(1-q^i)+(1-f)e^\theta]+(1-f)((1-q^i)e^\theta}}$$

………....(3.2)



Fig(1)

From the *W*-functions defined above, we obtain the probability that a unit is accepted and rejected respectively by sampling procedure as

$$[W_{1A}(\theta)]_{\theta=0} = q \quad \text{and}$$
$$[W_{1R}(\theta)]_{\theta=0} = p$$

Also, average number of units considered during a period of sampling inspection (v_1) is

$$v_1 = q[\frac{d}{d\theta} M_{LA}(\theta)]_{\theta=0} + p[\frac{d}{d\theta} M_{LR}(\theta)]_{\theta=0}$$
$$= 1/f$$

Where
$M_{LA}(\theta) = W_{LA}(\theta)/W_{LA}(0)$ **and**
$M_{LR}(\theta) = W_{LR}(\theta)/W_{LR}(0)$.

Therefore, average number of units passed during sampling inspection,
$v = (1/f).(1/p) = 1/fp$ and the average amount of inspection, $E(I)$, is

$$E(I) = \frac{1 - fq - (1-f)q^i}{fp + (1-f)pq^i}$$

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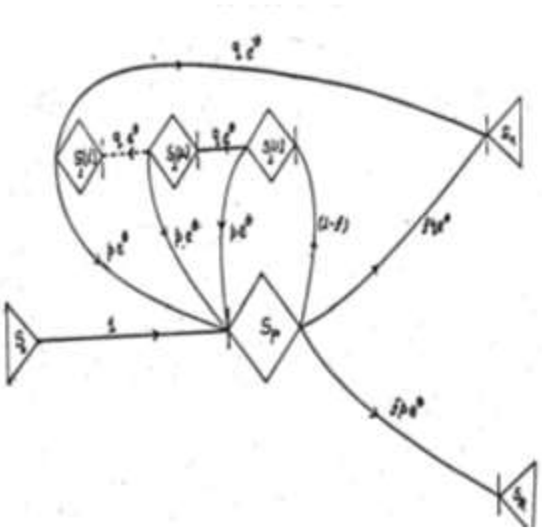

Fig(2)

The acceptance and rejection sequence of the CSP-1 inspection system during one inspection cycle can be represented[6] by Fig.(2). Consequently, the *W*-function from the initial node S_0 to the terminal S_A and S_R are respectively given as

$$W_A(\theta) = \frac{fqe^{\theta} + (1-f)(qe^{\theta})^i}{1 - [(1-f)pe^{\theta}[(1-(qe^{\theta})^i)/((1-qe^{\theta})]} \quad \ldots\ldots\ldots\ldots(3.3)$$

and

$$W_R(\theta) = \frac{fpe^{\theta}}{1 - [(1-f)pe^{\theta}[(1-(qe^{\theta})^i)/((1-qe^{\theta})]} \quad \ldots\ldots\ldots\ldots(3.4)$$

Therefore

$$P_A = [W_A(\theta)]_{\theta=0} = [fq + (1-f)q^i]/[f + (1-f)q^i] \ \ldots\ldots(3.5) \quad \text{and}$$

$$P_R = [W_R(\theta)]_{\theta=0} = fp/[f + (1-f)q^i](u+v) = f/[f + (1-f)q^i] \quad \ldots\ldots\ldots\ldots(3.6)$$

f = incoming quality at which AOQL occurs, then the following results due to Dodge (1943)

$$f = \frac{q_m^{i+1}}{i(AOQL)q_m^{i+1}} \quad \ldots\ldots\ldots\ldots(3.7)$$

where $q_w = 1 - p_w$

and $p_w = \frac{1+i(AOQL)}{i+1}$ ……….(3.8)

Thus, from equations (3.7) and (3.8), on simplification, we have

$$pq^{i-1}[i^2(AOQL) + iq_m^{i+1}(1+p)] - (1-q^i)[q_m^{i+1} + i(AOQL)q^i = 0 \ \ldots\ldots\ (3.9)$$

For given value of $p = p_w$ and AOQL, equation (3.8) can be solved for i by numerical methods and then the value of f can be found from equation (3.7).

Tables of CSP-1 Plan

This section provides a new approach for the selection of continuous sampling plan CSP 1plan using gert analysis technique. This section presents tables[4] to find a unique combination of plan parameters (i, f) that will achieve AOQL requirement and maximizes the average number of units inspected by CSP-1 plan during one inspection cycle $E(I)$, at $p = p_w$. The $E(I)$ function defined for one inspection cycle is maximum at $p = p_w$ when the system remains in the detailing state for a long time. Thus, $p = p_w$ is the worst incoming quality to be considered by the plan. Therefore, like AOQL the specification of p_w also alarms the state of corrective action to be taken by the producer.

Example

Suppose a CSP - 1 plan is required having AOQL = 0.05 and which maximizes $E(I)$ at pw = 0.12. Table (3.2.1) yields a CSP-1 plan with i = 48 and f = 0.0123.

TABLES OF i and f OF CSP-1 PLAN FOR GIVEN AOQL AND PROCE

AOQL=0.01		
p_w	i	f
0.0200	459	0.0008
0.0210	392	0.0018
0.0220	338	0.0036
0.0230	293	0.0065
0.0240	254	0.0101
0.0250	221	0.0176
0.0260	191	0.0270
0.0270	165	0.0404
0.0280	140	0.0600
0.0290	114	0.0923
0.0292	108	0.1022
0.0296	91	0.1373

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AOQL=0.02		
p_w	i	f
0.040	226	0.0008
0.042	193	0.0019
0.044	166	0.0037
0.046	144	0.0068
0.048	125	0.0113
0.05	109	0.0180
0.052	94	0.0273
0.054	82	0.0405
0.056	70	0.0589
0.058	58	0.0865
0.060	44	0.1410

AOQL=0.03		
p_w	i	f
0.055	198	0.0001
0.060	149	0.0009
0.065	115	0.0031
0.070	90	0.0083
0.075	72	0.0183
0.080	56	0.0385
0.085	44	0.0653
0.090	32	0.1201

AOQL=0.04		
p_w	i	f
0.08	109	0.0009
0.09	75	0.0054
010	53	0.0187
0.11	37	0.0485
0.12	25	0.1103
0.13	8	0.4194
0.14	6	0.5166
0.15	5	0.5498

AOQL=0.05		
p_w	i	f
0.11	63	0.0042
0.12	48	0.0123
013	36	0.0285
0.14	28	0.0571
0.15	20	0.1043
0.16	14	0.1825
0.17	10	0.2987
0.18	7	0.3877
0.19	6	0.4371
0.2	5	0.4693

Conclusion & suggestions

The work presented in this paper mainly relates construction and selection of tables for Continuous sampling plan CSP -1 using GERT analysis technique. GERT has been applied here to model and analyze the dynamics of the Dodge's CSP-1 plan. Procedures and tables has been provided to find a unique combination of (i, f) that will achieve the AOQL requirement and also, to optimize the average amount of inspection function, $E(I)$, when the process level $p = p_w$ is known. In CSP 1 plan, $p=p_w$ is the worst incoming quality to be considered by the plan. There, like AOQL the specification of p_w also alarms the state of corrective action to be taken by the producer. The study can be extended to CSP-2, CSP-3 and CSP-5 plans and also we can optimize all other sampling plans using GERT approach.

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